Ejercicios de cálculo de derivadas

1 Calcula las derivadas de las funciones:

1 $f(x)=5$

2 $f(x)=-2x$

3 $f(x)=-2x+2$

4 $f(x)=-2x^2-5$

5 $f(x)=2x^4+x^2-x^2+4$

6 $f(x)=\dfrac{x^3+2}{3}$

7 $f(x)=\dfrac{1}{3x^2}$

8 $f(x)=\dfrac{x+1}{x-1}$

9 $f(x)=\left(5x^2-3\right)\cdot\left(x^2+x+4\right)$

2 Calcula mediante la fórmula de la derivada de una potencia:

1 $f(x)=\dfrac{5}{x^5}$

2 $f(x)=\dfrac{5}{x^5}+\dfrac{3}{x^2}$

3 $f(x)=\sqrt{x}$

4 $f(x)=\dfrac{1}{\sqrt{x}}$

5 $f(x)=\dfrac{1}{x\sqrt{x}}$

6 $f(x)=\sqrt[3]{x^2}+\sqrt{x}$

7 $f(x)=\left(x^2+3x-2\right)^4$

3 Calcula mediante la fórmula de la derivada de una raíz:

1 $f(x)=\sqrt{x^2-2x+3}$

2 $f(x)=\sqrt[4]{x^5-x^3-2}$

3 $f(x)=\sqrt[3]{\dfrac{x^2+1}{x^2-1}}$

4 Deriva las funciones exponenciales

1 $f(x)=10^{\sqrt{x}}$

2 $f(x)=e^{3-x^2}$

3 $f(x)=\dfrac{e^x+e^{-x}}{2}$

4 $f(x)=3^{2x^2}\cdot\sqrt{x}$

5 $f(x)=\dfrac{e^{2x}}{x^2}$

5 Calcula la derivada de la funciones logarítmicas:

1 $f(x)=\ln\left(2x^4-x^3+3x^2-3x\right)$

2 $f(x)=\ln\left(\dfrac{e^x+1}{e^x-1}\right)$

3 $f(x)=\log\sqrt{\dfrac{1+x}{1-x}}$

4 $f(x)=\ln\sqrt{x\,(1-x)}$

5 $f(x)=\ln\sqrt[3]{\dfrac{3x}{x+2}}$

6 Calcula la derivada de la funciones trigonométricas:

1 $f(x)=sen\,\dfrac{1}{2}x$

2 $f(x)=\cos(7-2x)$

3 $f(x)=3tg2x$

4 $f(x)=\sec(5x+2)$

5 $f(x)=\sqrt[3]{senx}$

6 $f(x)=sen^3\ 3x$

7 $f(x)=cotg\,(3-2x)$

8 $f(x)=\cos\dfrac{x+1}{x-1}$

9 $f(x)=\sqrt{\dfrac{1-senx}{1+senx}}$

7 Calcula la derivada de la funciones trigonométricas inversas:

1 $f(x)=arc\ \ sen\left(1-2x^2\right)$

2 $f(x)=arc\ \ sen\sqrt{x^2-4}$

3 $f(x)=arc\ \ \cos e^x$

4 $f(x)=arc\ \ tg\sqrt{x}$

5 $f(x)=arctg\dfrac{1+x}{1-x}$

8 Derivar por la regla de la cadena las funciones:

1 $f(x) = \ln sen\, x$

2 $f(x) = \ln \cos 2x$

3 $f(x) = \ln tg\,(1 - x)$

4 $f(x) = \ln \sqrt{\dfrac{1 + sen\, x}{1 - sen\, x}}$

5 $f(x) = sen \sqrt{\ln (1 - 3x)}$

6 $f(x) = tg\left(sen\, \sqrt{5x}\right)$

7 $f(x) = sen^2 (\cos 2x)$

9 Deriva las funciones potenciales-exponenciales:

1 $f(x) = (senx)^{\cos x}$

2 $f(x) = \sqrt[x^2]{arc \cos x}$

3 $f(x) = \log_{sen\, x} x$

10 Hallar las derivadas sucesivas de:

1 $f(x) = 3x^4 + 5x^2 + 2x - 5$

2 $f(x) = \ln x$

3 $f(x) = sen\, x$

4 $f(x) = e^{-3x}$

11Derivar implicitamente:

1 $x^2 y - x y^2 + y^2 = 7$

2 $x^2 \operatorname{sen}(x + y) - 5y e^x = 3$

Soluciones:

1

Calcula las derivadas de las funciones:

1 $f(x) = 5$

$f'(x) = 0$

2 $f(x) = -2x$

$f'(x) = -2$

3 $f(x) = -2x + 2$

$f'(x) = -2$

4 $f(x) = -2x^2 - 5$

$f'(x) = -4x$

5 $f(x) = 2x^4 + x^3 - x^2 + 4$

$f'(x) = 8x^3 + 3x^2 - 2x$

6 $f(x) = \frac{x^3 + 2}{3}$

$f'(x) = x^2$

7 $f(x)=\frac{1}{3x^2}$

$$f'(x)=\frac{-6x}{(3x)^2}=\frac{-6x}{9x^4}=-\frac{2}{3x^3}$$

8 $f(x)=\frac{x+1}{x-1}$

$$f'(x)=\frac{1\cdot(x-1)-(x+1)\cdot 1}{(x-1)^2}=\frac{-2}{(x-1)^2}$$

9 $f(x)=\left(5x^2-3\right)\cdot\left(x^2+x+4\right)$

$$f'(x)=10x\left(x^2+x+4\right)+\left(5x^2-3\right)(2x+1)=20x^3+15x^2+34x-3$$

2

Calcula mediante la fórmula de la derivada de una potencia:

1 $f(x)=\frac{5}{x^5}=5x^{-5}$

$$f'(x)=-25x^{-6}=-\frac{25}{x^6}$$

2 $f(x)=\frac{5}{x^5}+\frac{3}{x^2}=5x^{-5}+3x^{-2}$

$$f'(x)=-25x^{-6}-6x^{-3}=-\frac{25}{x^6}-\frac{6}{x^3}$$

3 $f(x)=\sqrt{x}=x^{\frac{1}{2}}$

$$f'(x)=\frac{1}{2}x^{\frac{1}{2}-1}=\frac{1}{2}x^{-\frac{1}{2}}=\frac{1}{2\sqrt{x}}$$

4 $f(x)=\frac{1}{\sqrt{x}}=x^{-\frac{1}{2}}$

$$f'(x)=-\frac{1}{2}x^{-\frac{1}{2}-1}=-\frac{1}{2}x^{-\frac{3}{2}}=-\frac{1}{2\sqrt{x^3}}$$

5 $f(x)=\frac{1}{x\sqrt{x}}=\frac{1}{x\quad x^{\frac{1}{2}}}=x^{\frac{-3}{2}}$

$$f'(x)=\frac{-3}{2}x^{\frac{-5}{2}}=-\frac{3}{2\sqrt{x^5}}$$

6 $f(x)=\sqrt[3]{x^2}+\sqrt{x}=x^{\frac{2}{3}}+x^{\frac{1}{2}}$

$$f'(x)=\frac{2}{3}x^{\frac{2}{3}-1}+\frac{1}{2}x^{\frac{1}{2}-1}=\frac{2}{3}x^{-\frac{1}{3}}+\frac{1}{2}x^{-\frac{1}{2}}=\frac{2}{3\sqrt[3]{x}}+\frac{1}{2\sqrt{x}}$$

7 $f(x)=\left(x^2+3x-2\right)^4$

$$f'(x)=x^4\left(x^2+3x-2\right)^3(2x+3)$$

3

Calcula mediante la fórmula de la derivada de una raíz:

1 $f(x)=\sqrt{x^2-2x+3}$

$$f'(x)=\frac{2x-2}{2\sqrt{x^2-2x+3}}=\frac{x-1}{\sqrt{x^2-2x+3}}$$

2 $f(x)=\sqrt[4]{x^5-x^3-2}$

$$f'(x)=\frac{5x^4-3x^2}{4\sqrt[4]{\left(x^5-x^3-2\right)^3}}$$

3 $f(x)=\sqrt[3]{\frac{x^2+1}{x^2-1}}$

$$f'(x)=\frac{\frac{2x\left(x^2-1\right)-\left(x^2+1\right)2x}{\left(x^2-1\right)^2}}{3\sqrt[3]{\left(\frac{x^2+1}{x^2-1}\right)^2}}=$$

$$=\frac{\frac{-4x}{\left(x^2-1\right)^2}}{3\sqrt[3]{\left(\frac{x^2+1}{x^2-1}\right)^2}}=\frac{-4x}{3\left(x^2-1\right)^2\sqrt[3]{\left(\frac{x^2+1}{x^2-1}\right)^2}}=$$

$$\frac{-4x}{3\sqrt[3]{\left(\frac{x^2+1}{x^2-1}\right)^2\left(x^2-1\right)^4}}=\frac{-4x}{3\sqrt[3]{\left(x^2+1\right)^2\left(x^2-1\right)^2}}=$$

$$=\frac{-4x}{3\sqrt[3]{\left(x^4-1\right)^2}}$$

4

Deriva las funciones exponenciales:

1 $f(x)=10^{\sqrt{x}}$

$$f'(x)=\frac{1}{2\sqrt{x}}\cdot 10^{\sqrt{x}}\cdot \ln 10$$

2 $f(x)=e^{3-x^2}$

$$f'(x)=-2x\cdot e^{3-x^2}$$

3 $f(x)=\frac{e^{x}+e^{-x}}{2}$

$f'(x)=\frac{e^{x}-e^{-x}}{2}$

4 $f(x)=3^{2x^2}\cdot\sqrt{x}$

$f^{-1}(x)=4x\cdot3^{2x^2}\cdot\ln3\cdot\sqrt{x}+\frac{3^{2x^2}}{2\sqrt{x}}=$

$=3^{2x^2}\left(4x\cdot\sqrt{x}\cdot\ln3+\frac{1}{2\sqrt{x}}\right)$

5 $f(x)=\frac{e^{2x}}{x^2}$

$f'(x)=\frac{2\cdot e^{2x}\cdot x^2-e^{2x}\cdot2x}{x^4}=\frac{2x\cdot e^{2x}(x-1)}{x^4}=$

$=\frac{2\cdot e^{2x}(x-1)}{x^3}$

5

Calcula la derivada de la funciones logarítmicas:

1 $f(x)=\ln\left(2x^4-x^3+3x^2-3x\right)$

$f'(x)=\frac{8x^3-3x^2+6x-3}{2x^4-x^3+3x^2-3x}$

2 $f(x)=\ln\left(\frac{e^{x}+1}{e^{x}-1}\right)$

Aplicando las **propiedades de los logarítmos** obtenemos:

$f(x)=\ln\left(e^{x}+1\right)-\ln\left(e^{x}-1\right)$

$$f'(x)=\frac{e^x}{e^x+1}-\frac{e^x}{e^x-1}=\frac{e^{2x}-e^x-e^{2x}-e^x}{\left(e^x+1\right)\left(e^x-1\right)}=$$

$$=\frac{-2e^x}{e^{2x}-1}$$

3 $f(x)=\log\sqrt{\frac{1+x}{1-x}}$

Aplicando las **propiedades de los logarítmos** obtenemos:

$$f(x)=\frac{1}{2}\left[\log(1+x)-\log(1-x)\right]$$

$$f'(x)=\frac{1}{2}\left(\frac{1}{1+x}-\frac{-1}{1-x}\right)\cdot\log e=\frac{1}{2}\cdot\frac{1-x+1+x}{1-x^2}\cdot\log e=$$

$$=\frac{2}{1-x^2}\cdot\log e$$

4 $f(x)=\ln\sqrt{x(1-x)}$

Aplicando las **propiedades de los logarítmos** obtenemos:

$$f(x)=\frac{1}{2}\left[\ln x+\ln(1-x)\right]$$

$$f'(x)=\frac{1}{2}\left(\frac{1}{x}+\frac{-1}{1-x}\right)=\frac{1}{2}\cdot\frac{1-x-x}{x(1-x)}=$$

$$=\frac{1-2x}{2x(1-x)}$$

5 $f(x)=\ln\sqrt[3]{\frac{3x}{x+2}}$

Aplicando las **propiedades de los logarítmos** obtenemos:

$$f(x)=\frac{1}{3}\left[\ln 3x-\ln(x+2)\right]$$

$$f'(x)=\frac{1}{3}\left(\frac{3}{3x}-\frac{1}{x+2}\right)=\frac{1}{3}\cdot\frac{x+2-x}{x(x+2)}=$$

$$= \frac{2}{3x(x+2)}$$

6

Calcula la derivada de la funciones trigonométricas:

1 $f(x) = sen\,\frac{1}{2}x$

$$f'(x) = \frac{1}{2}\cos\frac{1}{2}x$$

2 $f(x) = \cos(7-2x)$

$$f'(x) = -(-2)\cdot sen(7-2x) = 2\cdot sen(7-2x)$$

3 $f(x) = 3\,tg\,2x$

$$f'(x) = 6\left(1+tg^2 2x\right)$$

4 $f(x) = \sec(5x+2)$

$$f'(x) = 5\;tg(5x+2)\cdot\sec(5x+2)$$

5 $f(x) = \sqrt[3]{senx}$

$$f'(x) = \frac{\cos x}{3\sqrt[3]{sen^2 x}}$$

6 $f(x) = sen^3\,3x$

$$f'(x) = 3\cdot sen^2\,3x\cdot 3\cdot\cos 3x = 9\cdot sen^2\,3x\cdot\cos 3x$$

7 $f(x) = cotg(3-2x)$

$$f'(x) = \frac{2}{sen^2(3-2x)}$$

8 $f(x)=\cos\frac{x+1}{x-1}$

$$f'(x)=-\frac{x-1-(x+1)}{(x-1)^2}\,sen\,\frac{x+1}{x-1}=\frac{2}{(x-1)^2}\cdot sen\,\frac{x+1}{x-1}$$

9 $f(x)=\sqrt{\frac{1-senx}{1+senx}}$

$$f'(x)=\frac{1}{2\sqrt{\frac{1-senx}{1+senx}}}\cdot\frac{-\cos x\,(1+senx)-(1+senx)\cos x}{(1+senx)^2}=$$

$$=\frac{1}{2\sqrt{\frac{1-senx}{1+senx}}}\cdot\frac{-\cos x-senx\cdot\cos x-\cos x+senx\cdot\cos x}{(1+senx)^2}=$$

$$=\frac{1}{2\sqrt{\frac{1-senx}{1+senx}}}\cdot\frac{-2\cos x}{(1+senx)^2}=\frac{-2\cos x}{2\sqrt{\frac{(1-senx)(1+senx)^4}{1+senx}}}=$$

$$=-\frac{\cos x}{\sqrt{(1-senx)(1+senx)^3}}=-\frac{\cos x}{\sqrt{(1-senx)(1+senx)(1+senx)^2}}=$$

$$=-\frac{\cos x}{\sqrt{1-senx}\cdot(1+senx)}=-\frac{\cos x}{\cos x\cdot(1+senx)}=$$

$$=-\frac{1}{1+senx}$$

7

Calcula la derivada de la funciones trigonométricas inversas:

1 $f(x)=arc\ sen\left(1-2x^2\right)$

$$f'(x)=\frac{-4x}{\sqrt{1-\left(1-2x^2\right)^2}}$$

2 $f(x)=arc\ sen\sqrt{x^2-4}$

$$f'(x)=\frac{1}{\sqrt{1-\left(x^2-4\right)}}\cdot\frac{2x}{2\sqrt{x^2-4}}=\frac{x}{\sqrt{5-x^2}\cdot\sqrt{x^2-4}}$$

3 $f(x)=arc\ \cos e^x$

$$f'(x)=-\frac{e^x}{\sqrt{1-e^{2x}}}$$

4 $f(x)=arc\ tg\sqrt{x}$

$$f'(x)=\frac{1}{1+x}\ \frac{1}{2\sqrt{x}}=\frac{1}{2\sqrt{x}\,(1+x)}$$

5 $f(x)=arctg\,\frac{1+x}{1-x}$

$$f'(x)=\frac{1}{1+\left(\frac{1+x}{1-x}\right)^2}\cdot\frac{1-x+1+x}{(1-x)^2}=$$

$$=\frac{1}{1+\frac{(1+x)^2}{(1-x)^2}}\cdot\frac{2}{(1-x)^2}=\frac{2}{(1-x)^2+(1+x)^2}=$$

$$=\frac{2}{1-2x+x^2+1+2x+x^2}=\frac{2}{2+2x^2}=$$

$$=\frac{1}{1+x^2}$$

8

Derivar por la regla de la cadena las funciones:

1 $f(x)=\ln sen\ x$

$$f'(x)=\frac{\cos x}{sen\ x}=cotg\,x$$

2 $f(x)=\ln\ \cos 2x$

$$f'(x)=\frac{-2sen\,2x}{\cos 2x}=-2\,tg\,2x$$

3 $f(x)=\ln\,tg\,(1-x)$

$$f'(x)=-\frac{1+tg^2\,(1-x)}{tg\,(1-x)}$$

4 $f(x)=\ln\sqrt{\dfrac{1+sen\,x}{1-sen\,x}}$

$$f(x)=\frac{1}{2}\left[\ln(1+sen\,x)-\ln(1-sen\,x)\right]$$

$$f'(x)=\frac{1}{2}\left(\frac{\cos x}{1+sen\,x}-\frac{-\cos x}{1-sen\,x}\right)=$$

$$=\frac{1}{2}\cdot\frac{\cos x-sen\,x\cos x+\cos x+sen\,x\cos x}{1-sen^2\,x}=$$

$$=\frac{1}{2}\cdot\frac{2\cos x}{\cos^2 x}=\frac{1}{\cos x}=\sec x$$

5 $f(x)=sen\sqrt{\ln(1-3x)}$

$$f'(x)=\cos\sqrt{\ln(1-3x)}\cdot\frac{1}{2\sqrt{\ln(1-3x)}}\cdot\frac{1}{1-3x}\cdot(-3)$$

6 $f(x)=tg\left(sen\sqrt{5x}\right)$

$$f'(x)=\left[1+tg^2\left(sen\sqrt{5x}\right)\right]\cdot\cos\sqrt{5x}\cdot\frac{1}{2\sqrt{5x}}\cdot 5$$

7 $f(x)=sen^2(\cos 2x)$

$$f'(x)=2\,sen\,(\cos 2x)\cdot\cos(\cos 2x)\cdot(-sen\,2x)\cdot 2$$

9

Deriva las funciones potenciales-exponenciales:

1 $f(x) = (senx)^{\cos x}$

$y = (senx)^{\cos x}$

$\ln y = \ln (senx)^{\cos x} \qquad \ln y = \cos x \ \ln (senx)$

$$\frac{y'}{y} = -senx \ \ln x + \cos x \frac{\cos x}{senx}$$

$$f'(x) = \left(-senx \ \ln x + \frac{\cos^2 x}{senx}\right)(senx)^{\cos x}$$

2 $f(x) = \sqrt[x^2]{arc\cos x}$

$y = (arc\cos x)^{\frac{1}{x^2}}$

$$\ln y = \frac{1}{x^2} \ \ln arc\cos x$$

$$\frac{y'}{y} = -\frac{2}{x^3} \ln arc\cos x - \frac{1}{x^2} \frac{1}{arc\cos x} \frac{1}{\sqrt{1-x^2}}$$

$$f'(x) = -\frac{1}{x^2} \ \sqrt[x^2]{arc\cos x}\left(\frac{2}{x} \ \ln arc\cos x + \frac{1}{\sqrt{1-x^2} \ \ arc\cos x}\right)$$

3 $f(x) = \log_{sen\,x} x$

$y = \log_{sen\,x} x \qquad (sen\,x)^y = x$

$\ln (sen\,x)^y = \ln x \qquad y \cdot \ln (sen\,x) = \ln x$

$$f(x) = \frac{\ln x}{\ln (sen\,x)}$$

$$f'(x) = \frac{1}{\ln^2(sen\, x)} \cdot \left(\frac{\ln(sen\, x)}{x} - \frac{\cos x}{sen\, x} \cdot \ln x \right) =$$

$$= \frac{1}{\ln^2(sen\, x)} \cdot \left(\frac{\ln(sen\, x)}{x} - cotg\, x \cdot \ln x \right)$$

10

Hallar las derivadas sucesivas de:

1 $f(x) = 3x^4 + 5x^2 + 2x - 5$

$f'(x) = 12x^3 + 10x^2 + 2$

$f''(x) = 36x^2 + 20x$

$f'''(x) = 72x + 20$

$f^{IV}(x) = 72$

$f^{V}(x) = 0$

2 $f(x) = \ln x$

$f'(x) = \frac{1}{x}$

$f''(x) = \frac{-1}{x^2}$

$f'''(x) = \frac{2}{x^3}$

$f^{IV}(x) = \frac{-2 \cdot 3}{x^4}$

...

$f^{n}(x) = (-1)^{n-1} \frac{(n-1)!}{x^n}$

3 $f(x) = sen\, x$

$$f'(x)=\cos x=sen\left(\frac{\pi}{2}+x\right)$$

$$f''(x)=-sen\,x=-\left[-sen\,(\pi+x)\right]=sen\left(2\cdot\frac{\pi}{2}+x\right)$$

$$f'''(x)=-\cos x=-sen\left(\frac{\pi}{2}+x\right)=-\left[-sen\left(3\cdot\frac{\pi}{2}+x\right)\right]=sen\left(3\cdot\frac{\pi}{2}+x\right)$$

...

$$f^{n}(x)=sen\left(\frac{n\cdot\pi}{2}+x\right)$$

3 $f(x)=e^{-3x}$

$$f'(x)=-3\cdot e^{-3x}$$

$$f''(x)=9\cdot e^{-3x}$$

$$f'''(x)=-27\cdot e^{-3x}$$

...

$$f^{n}(x)=(-3)^{n}\cdot e^{-3x}$$

11

Derivar implicitamente:

1 $x^2y-xy^2+y^2=7$

$$2xy+x^2y'-\left(y^2+2xyy'\right)+2yy'=0$$

$$2xy+x^2y'-y^2-2xyy'+2yy'=0$$

$$x^2y'-2xyy'+2yy'=-2xy+y^2$$

$$y'\left(x^2-2xy+2y\right)=y^2-2xy$$

$$y' = \frac{y^2 - 2x\,y}{x^2 - 2xy + 2y}$$

2 $x^2\, sen(x+y) - 5y\,e^x = 3$

$$y' = \frac{-\left[2x\ sen(x+y) + x^2 \cos(x+y) - 5y\,e^x\right]}{x^2 \cos(x+y) - 5e^x} =$$

$$y' = \frac{2x\ sen(x+y) + x^2 \cos(x+y) - 5y\,e^x}{-x^2 \cos(x+y) + 5e^x}$$